Cortona 2022

Dirac operators in Topology, Geometry and Representation Theory

Bernd Ammann (Regensburg)

Self-adjoint codimension 2 *boundary conditions for Dirac operators*

Abstract. Let N be an oriented compact submanifold of codimension 2 in an oriented complete Riemannian manifold M. We assume that $M \setminus N$ is spin and carries a unitary line bundle L. We study the self-adjoint extensions of the Dirac operator acting on spinors on $M \setminus N$, twisted by L. For the special case of $M = S^3$ with the round metric and flat connections on L, a special boundary condition within this framework was described in recent work by Portman, Sok and Solovej. It is said to provide magnetic link invariants for the link N and is motivated by stability of matter. By passing to a more geometric approach we are able to treat codimension 2 boundary conditions in any dimension and more general boundary conditions. Further, we obtain a classification of all self-adjoint boundary conditions. Possible self-adjoint boundary conditions are in bijection to Lagrangian subspaces in a space of sections of a bundle over N with carefully chosen Sobolev regularity.

Motivations for our work are a better understanding of associated link invariants, potential connections to the positive mass theorem for non-spin manifolds, and index theoretical methods for studying metrics of positive scalar curvature on non-spin manifold.

This project grew out of stimulating discussions with Boris Botvinnik and Nikolai Saveliev. Our work has a non-empty intersection with work by Albin, Leichtnam, Krainer, Mendoza and Piazza about stratified spaces, however in our special situation we have better control.

(Collaboration with N. Große, Freiburg)

Francesco Bei (Sapienza)

On weakly Kähler hyperbolic manifolds and geometric applications

Abstract. Kähler hyperbolic manifolds were introduced by Gromov around thirty years ago. In his seminal paper he showed that this class of Kähler manifolds enjoys several remarkable properties: for instance they are of general type, Kobayashi hyperbolic and their L^2 -Hodge numbers $h^{p,q}_{(2)}$ are positive if and only if $p = q$. In this talk I will report about a recent joint work with Simone Diverio, Philippe Eyssidieux, and Stefano Trapani where we introduced some weak versions of Kähler hyperbolicity. I will show how some of the properties proved by Gromov for Kähler hyperbolic manifolds remain true in our more general setting and I will explain various geometric applications. In particular I will describe how these ideas allow to verify some aspects of the Lang conjecture for Kähler hyperbolic manifolds.

Léo Bénard (Göttingen)

Ruelle zeta functions and Reidemeister-Turaev torsion on unit tangent bundles of hyperbolic surfaces

Abstract. Given a manifold endowed with a vector field and a unitary representation of its fundamental group, Fried conjectured that the value at zero of the dynamical zeta function of Ruelle was equal to a topological real-valued invariant: the R-torsion. Many cases of this conjecture has been established since then. With Jan Frahm and Polyxeni Spilioti (Aarhus), we study the case where the manifold M is the unit tangent bundle of a hyperbolic surface, possibly with singularities, together with the geodesic flow. In this setting, we show that the conjecture still holds true in a wider setting: for any representation of the fundamental group of M, we establish equality between the value at zero of the zeta function and a complex-valued invariant, the Reidemeister-Turaev torsion, refining the former R-torsion with the help of Turaev's "Euler structures".

Boris Botvinnik (Oregon)

*Spin^c-manifolds, psc-metrics and manifolds with fibered singularities.***Abstract.** I would like to discuss a problem of existence of positive scalar curvature on manifolds with fibered singularities. It turns out there are necessary and sufficient conditions for a psc-metric to exist on such objects. There is a particular case of manifolds with fibered singularities when the fiber is a circle. This case leads to psc-metrics on spin^c manifolds with special conditions near the singular locus. In particular, I describe some results concerning metrics on spin^c manifolds with positive "twisted scalar curvature", where the twisting comes from the curvature of the spin line bundle. One of the results here is that a simply-connected spin^c (not spin) manifold (M, L) of dimension at least five admits a positive twisted scalar curvature if and only if the index of the corresponding spin^c Dirac operator vanishes in the complex K-theory.

Simone Cecchini (Göttingen)

Metric inequalities with positive scalar curvature

Abstract. In recent years, distance inequalities on Riemannian manifolds under a positive scalar curvature bound have become an important aspect in the study of scalar curvature, in particular, due to many new ideas and conjectures raised by Gromov. In this talk, I will mostly focus on Riemannian bands, that is, Riemannian manifolds diffeomorphic to $N \times J$, where N is a closed manifold and J is an interval. In the first part, I will use a perturbation of the Dirac method to show that, if N is a closed, connected spin n-manifold with non-vanishing Rosenberg index and g is a metric on $N \times [-1, 1]$ whose scalar curvature is bounded from below by $n(n-1)$, then the distance between the two boundary components of $N \times [-1, 1]$ is at most $2\pi/n$. This gives, in many geometrically relevant cases, a sharp answer to a question raised by Gromov. In the second part of the talk, I will use a perturbation of the minimal hypersurface technique to show the following result. Let N be a closed, oriented n-manifold, with $n \leq 7$ and $n \neq 4$. If the cylinder $N \times (-\infty, \infty)$ carries a complete metric of positive scalar curvature, then N carries a metric of positive scalar curvature. This establishes, up to dimension 7, a conjecture due to Rosenberg and Stolz. The results using the Dirac method are joint work with Rudolf Zeidler. The results using the minimal hypersurface method are joint work with Daniel Räde and Rudolf Zeidler.

Nelia Charalambous (Cyprus)

The L^p spectrum of the Dirac operator.

Abstract. In this talk we will find sufficient conditions on a Clifford bundle over a Riemannian manifold so that the L^p spectrum of the Dirac operator and its square is independent of p for $p \geq 1$. Then, using the L^1-spectrum, which is simpler to compute, we generalize the class of manifolds over which the L^p-spectrum of the Dirac operator is the real line for all p. We also show that by applying the generalized Weyl criterion, we can find large classes of manifolds with asymptotically nonnegative Ricci curvature, or which are asymptotically flat, such that the L^2-spectrum of a general Dirac operator and its square is maximal. This joint work with Nadine Grosse.

Clément Cren (Créteil)

Transversal index for transversally Rockland operators on filtered manifolds.

Abstract. Usual transversal index theory allows, given an operator whose symbol is invertible in the directions transverse to the foliation, to construct a K-homology class for the C^* algebra of the holonomy groupoid of the foliation. This corresponds to an elliptic operator on the abstract "space of leaves of the foliation. We adapt this construction to filtered manifolds (i.e. a manifold whose tangent bundle is filtered by subbundles). Filtered manifolds come with a particular pseudodifferential calculus where the operators often fail to be elliptic, ellitpticity hence being replaced by the Rockland condition.

Given a foliated manifold with a filtration of the normal bundle, we define a notion of transversally Rockland symbols. We show that operators with transversally Rockland symbol yield a transverse K-homology class. We also create an equivariant KK-class from the symbol itself and establish a Poincaré duality type result between the two classes.

Georg Frenck (Augsburg)

Spaces of positive scalar curvature metrics on totally nonspin manifolds.

Abstract. In recent years, the Dirac operator on a spin manifold M has played a central role in the study of the space of positive scalar curvature metrics on M. In this talk, I will investigate, if it is possible to use index theory of the Dirac operator to study this space if the underlying manifold is totally nonspin, i.e. if its universal cover does not admit a spin structure. More precisely, I will answer the following 3 questions:

1. Can one use the Dirac-operator of a codimension 0 spin-submanifold to study the space of psc-metrics on M?
2. Can one use the Dirac-operator of the boundary to study the space of psc-metrics on a totally nonspin manifold M with spin boundary?
3. Are there ways to study the space of psc-metrics on M without employing Dirac-operators?

Jonathan Glöckle (Regensburg)

Enlargeability obstruction for spacetimes with both big bang and big crunch

Abstract. Enlargeability is a powerful obstruction to positive scalar curvature on spin manifolds. In this talk, I discuss an extension of this to the dominant energy condition (dec) for initial data sets. This extension is obtained by replacing the Dirac operator with a Dirac-Witten operator in the proof. The statement is the following: Within the space of dec initial data sets those with positive mean curvature cannot be connected by a path to those with negative mean curvature if the manifold is enlargeable and spin. In the context of general relativity, the result can be interpreted as obstruction to certain spacetimes with both big bang and big crunch singularity.

Sebastian Goette (Freiburg)

Extra twisted connected sums and their nu-invariants

Abstract. Riemannian Manifolds with holonomy G_2 are interesting both for geometers and for theoretical physicists. While many examples have been constructed, not much is known about the G_2 moduli space. The Crowley-Nordström ν-invariant and its analytic refinement $\bar{\nu}$ can be used to distinguish at least some of its connected components. Finally, we will sketch its computation for so-called extra twisted connected sums.

Thorsten Hertl (Göttingen)

Positive Scalar Curvature from a Concordance Viewpoint

Abstract. Scalar curvature is a local invariant of a Riemannian manifold. It measures asymptotically the volume growth of geodesic balls. Understanding the topological space of all positive scalar curvature metrics on a closed manifold has been an active field of study during the last 30 years. So far, these spaces have been considered from an isotopy viewpoint. I will describe a new approach to study this space based on the notion of concordance. To this end, I construct with the help of cubical set theory a comparison space that only encodes concordance information and in which the space of psc metrics canonically embeds. After the presentation of some of its properties, I will show that the indexdifference factors over the comparison space using a new model of real K-theory that is based on pseudo Dirac operators.

Peter Hochs (Radboud)

Equivariant analytic torsion for proper actions

Abstract. Analytic torsion for compact manifolds was constructed by Ray and Singer in the 1970s, as a way to realise Reidemeister-Franz torsion analytically. Cheeger and Mueller proved independently that the two notions of torsion are indeed equal. In this way, analytic torsion is a link between analysis and topology. It is also related to Quillen metrics, and to dynamical systems via the Fried conjecture. In the 1990s, different approaches to a construction of equivariant analytic torsion were developed, incorporating group actions. The two main kinds

of group actions considered were actions by finite or compact groups on compact manifolds, and actions by fundamental groups on universal covers of compact manifolds. With Hemanth Saratchandran, we construct a general notion of equivariant analytic torsion for proper group actions, and study its properties. This unifies and extends earlier work on equivariant analytic torsion.

Ursula Ludwig (Münster)

Bismut-Zhang theorem for singular spaces

Abstract. The famous theorem of Cheeger and Muller states the equality between the analytic (or Ray-Singer) torsion and the topological torsion of a smooth compact manifold equipped with a unitary flat vector bundle. Using local index techniques and the Witten deformation Bismut and Zhang gave the most general comparison theorem of torsions for a smooth compact manifold. The aim of this talk is the generalisation of the Bismut-Zhang theorem to the context of isolated conical singularities: We first establish a comparison formula between the analytic torsion and a torsion, which we call the Bismut-Zhang torsion. We also establish anomaly formulas for all three terms in the comparison formula, i.e. we study how the terms behave under variations of the Riemannian conical

Xiaonan Ma (Université Cité)

Differential K-theory and localization formula for eta invariants

Abstract. We establish a localization formula for equivariant eta invariants by combining our localization formula in differential K-theory with our extension of Goette's result on the comparison of two types of equivariant eta invariants. An important step in our approach is to construct a pre-λ-ring structure in differential K-theory.

Omar Mohsen (Paris-Saclay Université)

An introduction to maximally hypoelliptic differential operators.

Abstract. Maximally hypoelliptic differential operators are differential operators which enjoy regularity properties very similar to that of elliptic operators. Examples include elliptic operators and Hörmanders sum of squares. I will talk about recent developments concerning principal symbol and index theorem for maximally hypoelliptic differential operator. This is based on joint work with Androulidakis and Yuncken.

Werner Müller (Bonn)

Heat asymptotics and Weyl law for locally symmetric manifolds of finite volume

Abstract. This talk is concerned with spectral theory of Laplace type operators on non-compact locally symmetric manifolds X of finite volume. Such a manifold is the quotient of a

Riemannian symmetric space G/K of non-positive curvature by a discrete torsion free subgroup $\Gamma \subset G$. We consider Bochner-Laplace operators Δ_E acting in the space of smooth sections of a locally homogeneous vector bundle E over X. The spectrum of Δ_E is known to consist of a sequence of eigenvalues and a continuous spectrum which is governed by the boundary strata of the Borel-Serre compactification. Motivated by conjectural applications to number theroy, the existence and the size of the point spectrum is of particular interest. Most eigenvalues, if they exist, are embedded in the continuous spectrum, which makes it difficult to study them. We show that for arithmetic groups Γ, the heat operator restricted to the subspace spanned by the eigenfunctions is of trace class and the trace admits an asymptotic expansion for small time containing logarithmic terms and with leading term similar to the compact case. This implies the Weyl law for the eigenvalue counting function and shows that eigenvalues exist in abundance. The method is based on the Arthur-Selberg trace formula and the analysis of the scattering matrices.

Valerio Proietti (Shanghai)

Classification via index theory on the mapping torus

Abstract. Given a class of topological dynamical systems, we study the associated mapping torus from the point of view of foliated spaces. By studying the interaction between the leafwise Dirac operator and the invariant transverse measures, we completetely reframe in a geometric fashion the Elliott invariant for the crossed poduct of the dynamical system, and prove a rigidity result for the mapping torus, lifting leafwise homotopy equivalences to isomorphism of the noncommutative leaf space.

Frédéric Rochon (Montreal)

L^2-cohomology of quasi-fibered boundary metrics

Abstract. Quasi-fibered boundary metrics (QFB metrics) form a class of complete metrics generalizing the class of quasi-asymptotically locally Euclidean metrics introduced by Joyce. After reviewing what QFB metrics are and presenting natural examples, we will present a new approach to compute the L^2-cohomology of such metrics allowing to prove the Vafa-Witten conjecture and making advances on the Sen conjecture. This is based on a joint work with Chris Kottke.

Mehran Seyedhosseini (Potsdam)

Relative L^2-rho-invariants and the moduli space of positive scalar curvature metrics on manifolds with boundary

Abstract. The aim of this talk is to present some results on the topology of the moduli space of positive scalar curvature (psc) metrics on manifolds with boundary which are collared near the boundary. After a brief statement of known results for closed manifolds, I will define a relative L^2-rho-invariant associated to such metrics. I will then sketch how such invariants can be used

to show that if a 4k+3-dimensional spin manifold with boundary admits a psc metric that is collared near the boundary and a certain algebraic condition on the fundamental groups of the manifold and its boundary is satisfied, then the moduli space of psc metrics has infinitely many path components.

Shu Shen (Sorbonne)

Coherent sheaves, superconnection, and the Riemann-Roch-Grothendieck formula.

Abstract. In this talk, I will explain a construction of Chern character for coherent sheaves on a closed complex manifold with values in Bott-Chern cohomology. I will also show a corresponding Riemann-Roch-Grothendieck formula, which holds for general holomorphic maps between closed non-Kahler manifolds. Our proof is based on two fundamental objects : the superconnection and the hypoelliptic deformations. This is a joint work with J.-M. Bismut and Z. Wei arXiv:2102.08129.

Georges Skandalis (Université Paris Cité)

The Godbillon-Vey invariant in KK-theory with real coefficients

Abstract. Traces on C^*-algebras play an important role in index theory, for instance they can extract numerical invariants from index classes defined in K-theory. By introducing real coefficients in Kasparov bivariant K-theory (KK-theory), we show that traces on C^*-algebras become classes in $KK_{\mathbb{R}}$. The operation of applying a trace is then a Kasparov product. We investigate in particular a natural $KK_{\mathbb{R}}$-class representing the Godbillon–Vey invariant of a foliation of codimension 1. The Godbillon–Vey invariant deals with a (densely defined) infinite trace, and we will explain how such a trace gives a $KK_{\mathbb{R}}$-element. Joint work with Paolo Antonini and Sara Azzali.

Yanli Song (St. Louis, Missouri)

Higher APS index theorem for proper Lie group action

Abstract. In this talk, we will present an index theorem for proper cocompact Lie group actions on manifolds with boundary, which generalizes the Atiyah- Patodi-Singer index theorem for compact manifolds and the Atiyah-Bott fixed point index theorem for compact Lie group actions on closed manifolds. In addition, We give sufficient conditions ensuring the well-definedness of the delocalized eta invariant associated to a Dirac operator on a cocompact manifold without boundary. The talk is based on the joint work with Paolo Piazza, Hessel Posthuma and Xiang Tang.

Alexander Strohmaier (Leeds)

The eta invariant on curved spacetimes

Abstract. I will give a definition of the eta invariant that generalises to curved spacetimes and I will show that this reproduces the classical eta invariant of the Cauchy surface for simple ultrastatic spacetimes. This invariant appears as a boundary correction term in index problems for hyperbolic evolution equations. (Based on joint work with C. Baer)

Michał Wrochna (Cergy)

Dynamical residues of spectral zeta functions

Abstract. The Guillemin–Wodzicki residue density plays a distinguished role in theorems relating the spectral theory of elliptic operators with geometric invariants. In this talk, based on joint work with Nguyen Viet Dang (Sorbonne Université), I will give a generalization which refers to the dynamics of scaling towards the diagonal and which applies e.g. to hyperbolic operators arising in the Lorentzian context. This will serve as an opportunity to review the Hadamard parametrix and its variants, as well as Feynman propagators and Lorentzian spectral zeta function densities.

Vito Felice Zenobi (Roma)

Interior Kasparov products for rho-classes on Riemannian foliated bundles

Abstract. If we have a suitably oriented inclusion of the foliation $\mathcal{F}$ into the foliation $\mathcal{F}'$ over a closed manifold M, then we extend the construction of the lower shriek maps given by Hilsum and Skandalis to adiabatic deformation groupoid C^*-algebras: we construct an asymptotic morphism from the adiabatic deformation of G to the adiabatic deformation of H where G and H are the monodromy groupoids associated with $\mathcal{F}$ and $\mathcal{F}'$ respectively. Furthermore, we prove an interior Kasparov product formula for foliated rho-classes associated with longitudinal metrics of positive scalar curvature in certain simple cases.